Introduction to Formal Analysis
Educational subject description sheet

Basic information

Study programme Matematyka	**Didactic cycle** 2025/26
Speciality -	**Subject code** 06MATS.22K.11710.25
Organizational unit Faculty of Mathematics and Computer Sciences	**Lecture languages** English
Study level Second-cycle programme	**Course type** Elective
Study form Full-time	**Block** Major subjects
Education profile General academic	

Subject coordinator	Dariusz Bugajewski
Lecturer	Dariusz Bugajewski

Period Semester 2	**Activities and hours** • Lecture: 30, Exam • Classes: 30, Graded credit	**Number of ECTS points** 6

Goals

Code	Goal
C1	The main goal of Formal Analysis is to investigate formal power series and formal Laurent series as well. Various applications of that series ranging from algebra to classical analysis. In particular, it is worth to mention here that it has some applications to the investigation of the boundary convergence behavior of power series. The main goal of this course is to introduce basic notions and tools using in Formal Analysis as well as some of its applications. A particular emphasis will be put on some new achievements of that theory.

Entry requirements

One has to have basic knowledge in mathematical analysis. Moreover, some knowledge of basic definitions and facts of functional analysis and metric topology would be also useful.

Subject learning outcomes

Code	Outcomes in terms of	Learning outcomes	Examination methods
Knowledge – Student:			
W1	knows basic definitions and algebraic properties of formal power series.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W2	knows the composition of a formal power series with a nonunit.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W3	knows Right Distributive Law for composition.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W4	knows the definition of the general composition of formal power series. Knows the necessary and sufficient conditions for the existence of such a composition.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W5	knows the General Right Distributive Law for the composition.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W6	knows the basic facts of the calculus of formal power series, in particular, the definition of the formal derivative and the Generalized Chain Rule.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W7	knows applications of formal analysis to boundary convergence of regular power series.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W8	knows the way of defining of the topology on the space of formal power series.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W9	knows the definition of formal Laurent series and their basic algebraic properties.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W10	knows the definition of the composition of formal Laurent series and formal power series and the conditions for the existence such a composition.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
W11	knows the way of defining of the topology on the space of formal Laurent series.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Written colloquium
Skills – Student:			
U1	can find the product of formal power series.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Written colloquium
U2	can determine the existence the composition of given formal power series.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Written colloquium

Code	Outcomes in terms of	Learning outcomes	Examination methods
U3	can find the formal derivative of formal power series, in particular, applying the Generalized Chain Rule.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Written colloquium
U4	can apply formal analysis to examination of boundary convergence of regular power series.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Written colloquium
U5	can apply fixed point theorems to mappings defined on the space of formal power series.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written colloquium
U6	can find the product of formal Laurent series.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Written colloquium
U7	can determine the existence the composition of given formal power series and formal Laurent series.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written colloquium
U8	can apply fixed point theorems to mappings defined on the space of formal Laurent series.	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written colloquium

Study content

No.	Course content	Subject learning outcomes	Activities
1.	Definitions and algebraic properties of formal power series.	W1, U1	Lecture, Classes
2.	The composition of a formal power series with a nonunit.	W2, U1, U2	Lecture, Classes
3.	Right Distributive Law for composition.	W3, U2	Lecture, Classes
4.	General composition of formal power series.	W4, U1, U2	Lecture, Classes
5.	General Right Distributive Law for composition.	W3, W5, U1, U2	Lecture, Classes
6.	Calculus of formal power series.	W6, U3	Lecture, Classes
7.	The Generalized Chain Rule.	W6, U3	Lecture, Classes
8.	Boundary convergence of regular power series.	W7, U4	Lecture, Classes
9.	Topology on the space of formal power series.	W8, U5	Lecture, Classes
10.	Basic algebra of formal Laurent series.	W9, U6	Lecture, Classes
11.	Composition of formal Laurent series and formal power series.	W10, U6, U7	Lecture, Classes
12.	Topology on the space of formal Laurent series.	W11, U8	Lecture, Classes

Additional information

Activities	Teaching and learning methods and activities
Lecture	Lecture with a multimedia presentation of selected issues
Classes	Solving tasks (e.g. computational, artistic, practical), Classes method, Activating method - "brainstorming"

Activities	Credit conditions
Lecture	The condition for taking the exam (written form) is to obtain passing grades in exercises. Grading scale with applied percentage distribution: excellent (5.0): achievement of the student's expected learning outcomes at a minimum of 90.0%. very good (4.5): achievement by the student of the desired learning outcomes ranging from 80.0% - 89.9%. good (4.0): achievement of student learning outcomes 70.0% - 79.9%. average (3.5): achievement of student learning outcomes 60.0% - 69.9%. satisfactory (3.0): attainment of the student learning outcomes within 50.0% - 59.9%. unsatisfactory (2.0): failure of the student to achieve the expected learning outcomes below 50.0%.
Classes	Written colloguium. Grading scale with applied percentage distribution: excellent (5.0): achievement of the student's expected learning outcomes at a minimum of 90.0%. very good (4.5): achievement by the student of the desired learning outcomes ranging from 80.0% - 89.9%. good (4.0): achievement of student learning outcomes 70.0% - 79.9%. average (3.5): achievement of student learning outcomes 60.0% - 69.9%. satisfactory (3.0): attainment of the student learning outcomes within 50.0% - 59.9%. unsatisfactory (2.0): failure of the student to achieve the expected learning outcomes below 50.0%.

Literature

Obligatory

1. X. -X. Gan, Selected Topics of Formal Analysis, Lecture Notes in Nonlinear Analysis, Lecture Notes in Nonlinear Analysis, vol. 15, Juliusz Schauder Center for Nonlinear Studies, 2017

Optional

1. D. Bugajewski and X. -X. Gan, A note on formal power series, Commentationes Mathematicae Universitatis Caroline 51(4)(2010), 595-604
2. D. Bugajewski and X. -X. Gan, On formal Laurent series, Bulletin of the Brazilian Mathematical Society, New Series 42(3)(2011), 415-437

Calculation of ECTS points

Activities	Activity hours*
Lecture	30
Classes	30
Preparation for the exam	45

Preparation for the assessment	30
Preparation for classes	20
Other	25

Student workload	**Hours** 180
Number of ECTS points	**ECTS** 6

* academic hour = 45 minutes

Efekty uczenia się dla kierunku

Kod	Treść
MAT_K2_U02	The graduate can przeprowadzać rozumowania matematyczne, dowodzenie twierdzeń, jak i weryfikację hipotez drogą doboru odpowiednich przykładów
MAT_K2_U07	The graduate can posługiwać się narzędziami i aparatem analizy matematycznej oraz zna jej znaczenie i zastosowanie w poznanych działach matematyki
MAT_K2_U09	The graduate can posługiwać się zaawansowanymi metodami i narzędziami przynajmniej z jednej dziedziny matematyki
MAT_K2_U10	The graduate can komunikować się w co najmniej jednym języku obcym na poziomie średniozaawansowanym B2+ z uwzględnieniem języka specjalistycznego z zakresu matematyki
MAT_K2_W01	The graduate knows and understands klasyczne pojęcia z zakresu matematyki i jej zastosowań oraz najważniejsze metody i twierdzenia z głównych jej działów
MAT_K2_W02	The graduate knows and understands rolę, znaczenie i zasady poprawnego prowadzenia rozumowań matematycznych oraz zna różne techniki dowodzenia
MAT_K2_W04	The graduate knows and understands specjalistyczne zagadnienia z wybranej dziedziny matematyki