# Introduction to Formal Analysis
## Educational subject description sheet

## Basic information

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| **Study programme** <br> Matematyka | **Didactic cycle** <br> 2024/25 |
| **Speciality** <br> - | **Subject code** <br> 06MATS.22K.11710.24 |
| **Organizational unit** <br> Faculty of Mathematics and Computer Sciences | **Lecture languages** <br> English |
| **Study level** <br> Second-cycle programme | **Course type** <br> Elective |
| **Study form** <br> Full-time | **Block** <br> Major subjects |
| **Education profile** <br> General academic | |

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| **Subject coordinator** | Dariusz Bugajewski |

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| **Lecturer** | Dariusz Bugajewski |

| Period <br> Semester 2 | Activities and hours <br> • Lecture: 30, Exam <br> • Classes: 30, Graded credit | Number of ECTS points <br> 6 |
|---|---|---|

## Goals

| Code | Goal |
|---|---|
| C1 | The main goal of Formal Analysis is to investigate formal power series and formal Laurent series as well. Various applications of that series ranging from algebra to classical analysis. In particular, it is worth to mention here that it has some applications to the investigation of the boundary convergence behavior of power series. The main goal of this course is to introduce basic notions and tools using in Formal Analysis as well as some of its applications. A particular emphasis will be put on some new achievements of that theory. |

# Entry requirements

One has to have basic knowledge in mathematical analysis. Moreover, some knowledge of basic definitions and facts of functional analysis and metric topology would be also useful.

# Subject learning outcomes

| Code | Outcomes in terms of | Learning outcomes | Examination methods |
|---|---|---|---|
| **Knowledge – Student:** | | | |
| W1 | knows basic definitions and algebraic properties of formal power series. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W2 | knows the composition of a formal power series with a nonunit. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W3 | knows Right Distributive Law for composition. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W4 | knows the definition of the general composition of formal power series. Knows the necessary and sufficient conditions for the existence of such a composition. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W5 | knows the General Right Distributive Law for the composition. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W6 | knows the basic facts of the calculus of formal power series, in particular, the definition of the formal derivative and the Generalized Chain Rule. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W7 | knows applications of formal analysis to boundary convergence of regular power series. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W8 | knows the way of defining of the topology on the space of formal power series. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W9 | knows the definition of formal Laurent series and their basic algebraic properties. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W10 | knows the definition of the composition of formal Laurent series and formal power series and the conditions for the existence such a composition. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| W11 | knows the way of defining of the topology on the space of formal Laurent series. | MAT_K2_W01, MAT_K2_W02, MAT_K2_W04 | Written exam, Written colloquium |
| **Skills – Student:** | | | |
| U1 | can find the product of formal power series. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written exam, Written colloquium |
| U2 | can determine the existence the composition of given formal power series. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written exam, Written colloquium |

| Code | Outcomes in terms of | Learning outcomes | Examination methods |
|---|---|---|---|
| U3 | can find the formal derivative of formal power series, in particular, applying the Generalized Chain Rule. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written exam, Written colloquium |
| U4 | can apply formal analysis to examination of boundary convergence of regular power series. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written exam, Written colloquium |
| U5 | can apply fixed point theorems to mappings defined on the space of formal power series. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written colloquium |
| U6 | can find the product of formal Laurent series. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written exam, Written colloquium |
| U7 | can determine the existence the composition of given formal power series and formal Laurent series. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written colloquium |
| U8 | can apply fixed point theorems to mappings defined on the space of formal Laurent series. | MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10 | Written colloquium |

## Study content

| No. | Course content | Subject learning outcomes | Activities |
|---|---|---|---|
| 1. | Definitions and algebraic properties of formal power series. | W1, U1 | Lecture, Classes |
| 2. | The composition of a formal power series with a nonunit. | W2, U1, U2 | Lecture, Classes |
| 3. | Right Distributive Law for composition. | W3, U2 | Lecture, Classes |
| 4. | General composition of formal power series. | W4, U1, U2 | Lecture, Classes |
| 5. | General Right Distributive Law for composition. | W3, W5, U1, U2 | Lecture, Classes |
| 6. | Calculus of formal power series. | W6, U3 | Lecture, Classes |
| 7. | The Generalized Chain Rule. | W6, U3 | Lecture, Classes |
| 8. | Boundary convergence of regular power series. | W7, U4 | Lecture, Classes |
| 9. | Topology on the space of formal power series. | W8, U5 | Lecture, Classes |
| 10. | Basic algebra of formal Laurent series. | W9, U6 | Lecture, Classes |
| 11. | Composition of formal Laurent series and formal power series. | W10, U6, U7 | Lecture, Classes |
| 12. | Topology on the space of formal Laurent series. | W11, U8 | Lecture, Classes |

# Additional information

| Activities | Teaching and learning methods and activities |
|---|---|
| Lecture | Lecture with a multimedia presentation of selected issues |
| Classes | Solving tasks (e.g. computational, artistic, practical), Classes method, Activating method - "brainstorming" |

| Activities | Credit conditions |
|---|---|
| Lecture | The condition for taking the exam (written form) is to obtain passing grades in exercises. Grading scale with applied percentage distribution: excellent (5.0): achievement of the student's expected learning outcomes at a minimum of 90.0%. very good (4.5): achievement by the student of the desired learning outcomes ranging from 80.0% - 89.9%. good (4.0): achievement of student learning outcomes 70.0% - 79.9%. average (3.5): achievement of student learning outcomes 60.0% - 69.9%. satisfactory (3.0): attainment of the student learning outcomes within 50.0% - 59.9%. unsatisfactory (2.0): failure of the student to achieve the expected learning outcomes below 50.0%. |
| Classes | Written colloquium. Grading scale with applied percentage distribution: excellent (5.0): achievement of the student's expected learning outcomes at a minimum of 90.0%. very good (4.5): achievement by the student of the desired learning outcomes ranging from 80.0% - 89.9%. good (4.0): achievement of student learning outcomes 70.0% - 79.9%. average (3.5): achievement of student learning outcomes 60.0% - 69.9%. satisfactory (3.0): attainment of the student learning outcomes within 50.0% - 59.9%. unsatisfactory (2.0): failure of the student to achieve the expected learning outcomes below 50.0%. |

# Literature

**Obligatory**

1. X. -X. Gan, Selected Topics of Formal Analysis, Lecture Notes in Nonlinear Analysis, Lecture Notes in Nonlinear Analysis, vol. 15, Juliusz Schauder Center for Nonlinear Studies, 2017

**Optional**

1. D. Bugajewski and X. -X. Gan, A note on formal power series, Commentationes Mathematicae Universitatis Carolinae 51(4)(2010), 595-604
2. D. Bugajewski and X. -X. Gan, On formal Laurent series, Bulletin of the Brazilian Mathematical Society, New Series 42(3)(2011), 415-437

# Calculation of ECTS points

| Activities | Activity hours* |
|---|---|
| Lecture | 30 |
| Classes | 30 |
| Preparation for the exam | 45 |

| Preparation for the assessment | 30 |
|---|---|
| Preparation for classes | 20 |
| Other | 25 |

| **Student workload** | **Hours**<br>180 |
|---|---|
| **Number of ECTS points** | **ECTS**<br>6 |

\* academic hour = 45 minutes

# Efekty uczenia się dla kierunku

| Kod | Treść |
|---|---|
| MAT_K2_U02 | The graduate can przeprowadzać rozumowania matematyczne, dowodzenie twierdzeń, jak i weryfikację hipotez drogą doboru odpowiednich przykładów |
| MAT_K2_U07 | The graduate can posługiwać się narzędziami i aparatem analizy matematycznej oraz zna jej znaczenie i zastosowanie w poznanych działach matematyki |
| MAT_K2_U09 | The graduate can posługiwać się zaawansowanymi metodami i narzędziami przynajmniej z jednej dziedziny matematyki |
| MAT_K2_U10 | The graduate can komunikować się w co najmniej jednym języku obcym na poziomie średniozaawansowanym B2+ z uwzględnieniem języka specjalistycznego z zakresu matematyki |
| MAT_K2_W01 | The graduate knows and understands klasyczne pojęcia z zakresu matematyki i jej zastosowań oraz najważniejsze metody i twierdzenia z głównych jej działów |
| MAT_K2_W02 | The graduate knows and understands rolę, znaczenie i zasady poprawnego prowadzenia rozumowań matematycznych oraz zna różne techniki dowodzenia |
| MAT_K2_W04 | The graduate knows and understands specjalistyczne zagadnienia z wybranej dziedziny matematyki |