UNIWERSYTET IM. ADAMA MICKIEWICZA W POZNANIU

Elements of Applied Mathematics

Educational subject description sheet

Basic information

Study programme Matematyka **Speciality** - **Organizational unit** Faculty of Mathematics and Computer Sciences **Study level** Second-cycle programme **Study form** Full-time **Education profile** General academic	**Didactic cycle** 2023/24 **Subject code** 06MATS.22K.11709.23 **Lecture languages** English **Course type** Elective **Block** Major subjects

Subject coordinator	Daria Bugajewska
Lecturer	Daria Bugajewska

Period	**Activities and hours**	**Number of ECTS points**
Semester 2	• Lecture: 30, Exam • Classes: 30, Graded credit	6

Goals

Code	Goal
C1	The main goal of Elements of Applied Mathematics is to present some applications of nonlinear analysis, especially of ordinary differential equations. In particular, mathematical models involving first order differential equations as well as some applications of linear differential equations and systems of linear differential equations will be discussed. Discrete models will also be considered. Moreover, the Frobenius method of solving higher order differential equations will be presented along with its applications.

Subject learning outcomes

Code	Outcomes in terms of	Learning outcomes	Examination methods
Knowledge – Student:			
W1	knows basic definitions of solutions to ordinary differential equations.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Test
W2	knows basic existence and uniqueness theorems in differential equations theory.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Test
W3	knows the notions of direction fields, phase portrait (phase line) and types of equilibrium points.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Test
W4	knows some basic continuous models (as Malthusian Model, Logistic Model, Alle effects, dealyed logistic equation, Volterra-Lotka Model)	MAT_K2_W01, MAT_K2_W02, MAT_K2_W03, MAT_K2_W04	Written exam, Test
W5	knows definitions of linear and nonlinear difference equations, stationary points and their stability	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Test
W6	knows examples of discrete models (Model for an insect population, Model for Red Blood Cell (RBC) Production, Descrete Logistic equation)	MAT_K2_W01, MAT_K2_W02, MAT_K2_W03, MAT_K2_W04	Written exam
W7	knows Cauchy-Euler equidimensional equations.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Test
W8	knows the method of Frobenius.	MAT_K2_W01, MAT_K2_W02, MAT_K2_W04	Written exam, Test
W9	nows other applications of linear differential equations and systems of linear differential equations (e.g. heating and cooling of buildings, the mass-spring oscillator, free mechanical vibrations)	MAT_K2_W01, MAT_K2_W02, MAT_K2_W03, MAT_K2_W04	Written exam
Skills – Student:			
U1	can determine whether a given function is a solution to differential equation or whether it is a unique solution	MAT_K2_U01, MAT_K2_U02, MAT_K2_U05, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test
U2	can sketch direction fields, phase lines and isoclines for given equations	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test
U3	can determine equilibrium points and their stability	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test

Code	Outcomes in terms of	Learning outcomes	Examination methods
U4	can model simple phenomena and characterize behaviour of solutions of those models in the continuous case	MAT_K2_U02, MAT_K2_U06, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test
U5	can model simple phenomena and characterize behaviour of solutions of those models in the discrete case	MAT_K2_U02, MAT_K2_U06, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test
U6	can determine ordinary and singular points of differential equations of second order with analytic coefficients	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test
U7	can solve Cauchy-Euler (equidimensional) equations	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test
U8	can solve differential equations by using power series method (Frobenius method)	MAT_K2_U02, MAT_K2_U07, MAT_K2_U09, MAT_K2_U10	Written exam, Test

Study content

No.	Course content	Subject learning outcomes	Activities
1.	Basic existence and uniqueness theorems in differential equations theory.	W1, W2, U1	Lecture, Classes
2.	Direction fields, phase lines, types of equilibrium points.	W3, U2	Lecture, Classes
3.	Malthusian and continuous logistic models.	W4, U3, U4	Lecture, Classes
4.	Alle effects.	W4	Lecture
5.	Delayed logistic equation.	W4, U3, U4	Lecture, Classes
6.	Volterra-Lotka model.	W4	Lecture
7.	Definitions of linear and nonlinear difference equations, stationary points an their stability	W5, U3	Lecture, Classes
8.	Discrete Model for an insect population	W6, U3, U5	Lecture, Classes
9.	Discrete Model for Red Blood Cell (RBC) Production	W6, U3, U5	Lecture, Classes
10.	Discrete Logistic equation	W6, U3, U5	Lecture, Classes
11.	Equations with analytic coefficients	W8, U6	Lecture, Classes
12.	Cauchy-Euler equidimensional equations.	W7, U7	Lecture, Classes
13.	Method of Frobenius.	W8, U8	Lecture, Classes
14.	Heating and cooling of buildings.	W9	Lecture
15.	The mass-spring oscillator.	W9	Lecture

No.	Course content	Subject learning outcomes	Activities
16.	Free mechanical vibrations.	W9	Lecture

Additional information

Activities	Teaching and learning methods and activities
Lecture	Lecture with a multimedia presentation of selected issues
Classes	Solving tasks (e.g. computational, artistic, practical), Activating method - "brainstorming", Solving calculation tasks

Activities	Credit conditions
Lecture	A necessary condition to take the exam is a positive grade from the class. The exam will take 90 minutes. It will consists of two parts. The first part will concern theoretical knowledge (definitions, facts, theorems along with their proofs) - maximum 60 pointsThe second part will consist of some problems to solve - maximum 40 points. The minimal score needed to pass the exam is 50 points. Students who do not obtain 50 points may take a make-up exam. The final grade depends on the total number of points. Grading scale with applied percentage distribution: excellent (5.0): achievement of the student's expected learning outcomes at a minimum of 90.0%. very good (4.5): achievement by the student of the desired learning outcomes ranging from 80.0% - 89.9%. good (4.0): achievement of student learning outcomes 70.0% - 79.9%. average (3.5): achievement of student learning outcomes 60.0% - 69.9%. satisfactory (3.0): attainment of the student learning outcomes within 50.0% - 59.9%. unsatisfactory (2.0): failure of the student to achieve the expected learning outcomes below 50.0%.
Classes	There will be two tests during the semester. On each, one can get maximum 50 points. In case of an excused absence during the test, one is entitled to take the test at a later time agreed upon with the lecturer. In the case of an insufficient score one may take a make-up test covering the whole material presented during the semester. On this test one can obtain maximum 100 points. Grading scale with applied percentage distribution: excellent (5.0): achievement of the student's expected learning outcomes at a minimum of 90.0%. very good (4.5): achievement by the student of the desired learning outcomes ranging from 80.0% - 89.9%. good (4.0): achievement of student learning outcomes 70.0% - 79.9%. average (3.5): achievement of student learning outcomes 60.0% - 69.9%. satisfactory (3.0): attainment of the student learning outcomes within 50.0% - 59.9%. unsatisfactory (2.0): failure of the student to achieve the expected learning outcomes below 50.0%.

Literature

Obligatory

1. J. D. Murray, Mathematical Biology, Springer-Verlag, Berlin, 1989.
2. R. Nagle, E. Saff, A. Snider, Fundamentals of Differential Equations, Pearson, 2012.

Optional

1. J. Muller, C. Kittler, Methods and Models in Mathematical Biology: Deterministic and Stochastic Approaches , Lecture Notes on Mathematical Modelling in the Life Sciences, Springer, 2015

Calculation of ECTS points

Activities	**Activity hours***
Lecture	30
Classes	30
Preparation for the exam	40
Preparation for classes	20
Preparation for the assessment	35
Other	25
Student workload	**Hours** 180
Number of ECTS points	**ECTS** 6

* academic hour = 45 minutes

Efekty uczenia się dla kierunku

Kod	Treść
MAT_K2_U01	The graduate can wyrażać treści matematycznych w mowie i piśmie, w opracowaniach o różnym charakterze, dostosowując precyzję sformułowań i języka do poziomu i potrzeb odbiorcy opracowania
MAT_K2_U02	The graduate can przeprowadzać rozumowania matematyczne, dowodzenie twierdzeń, jak i weryfikację hipotez drogą doboru odpowiednich przykładów
MAT_K2_U05	The graduate can analizować nowe zagadnienia, korzystać z literatury matematycznej, baz danych i innych źródeł oraz dokonać krytycznej ich oceny
MAT_K2_U06	The graduate can odnosić pojęcia matematyczne do niematematycznych kontekstów, w analizowanych problemach potrafi dostrzec i wykorzystać struktury formalne opisywane w wybranych działach matematyki
MAT_K2_U07	The graduate can posługiwać się narzędziami i aparatem analizy matematycznej oraz zna jej znaczenie i zastosowanie w poznanych działach matematyki
MAT_K2_U09	The graduate can posługiwać się zaawansowanymi metodami i narzędziami przynajmniej z jednej dziedziny matematyki
MAT_K2_U10	The graduate can komunikować się w co najmniej jednym języku obcym na poziomie średniozaawansowanym B2+ z uwzględnieniem języka specjalistycznego z zakresu matematyki
MAT_K2_W01	The graduate knows and understands klasyczne pojęcia z zakresu matematyki i jej zastosowań oraz najważniejsze metody i twierdzenia z głównych jej działów
MAT_K2_W02	The graduate knows and understands rolę, znaczenie i zasady poprawnego prowadzenia rozumowań matematycznych oraz zna różne techniki dowodzenia
MAT_K2_W03	The graduate knows and understands podstawy konstruowania modeli matematycznych przydatnych w zastosowaniach matematyki w różnych dziedzinach wiedzy
MAT_K2_W04	The graduate knows and understands specjalistyczne zagadnienia z wybranej dziedziny matematyki